CCSSM:

Keeping Teaching and Learning Strong

How might we use the Common Core State Standards for Mathematics to keep the deep learning of every student at the center of our work?

By Susan Jo Russell

To support mathematics educators as they consider implications of the Common Core State Standards for Mathematics (CCSSM) for instruction and assessment, *Teaching Children Mathematics* launched a series of articles beginning in the February 2012 issue. In this concluding installment, we concentrate on the implementation of the eight Standards of Mathematical Practice and the constellations of Practices and Standards. In the September issue, Matthew Larson follows up the series with a feature article that looks at CCSSM through the lens of mathematics education reform history and asks the provocative question, Will CCSSM Matter in Ten Years?

The Common Core State Standards in Mathematics (CCSSM) are, at the same time, promising and problematic. Some educators hope that the adoption and implementation of this document can result in a deeper, more coherent curriculum for all students. Others are concerned that it will push schools and teachers to be even more focused than they have been on high-stakes tests and that it will be implemented as a list of items to "cover" rather than as a lattice on which strong teaching and learning must be woven.

Like any set of standards, CCSSM does not arise from some infallible experiment. The document was written by humans wrestling to accommodate a variety of strong and contradictory opinions about what mathematics should be taught. It is a product of argument and compromise, written to meet unrealistic timelines. And, as the document itself points out, while existing research and "best practices" were considered, "there is more to be learned about the

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most essential knowledge for student success" (CCSSI 2010 Introduction).

The adoption of CCSSM by most states and the power it wields because it is tied to federal dollars and to high-stakes assessment, presents us, then, with a huge responsibility—to mine these Standards deeply for opportunities they can offer for the teaching and learning of mathematics. These Standards, which will govern many, many classrooms and students for some time to come, do not lay out our teaching for us. They are not a curriculum. They are not a sequence of instruction. They cannot, and should not, be translated, standard by standard, into a set of lesson plans. They do not illuminate how to support the student who is struggling with grade-level computation while providing an appropriate level of challenge and engagement for every student in the class. The CCSSM document makes this point: "These Standards do not dictate curriculum or teaching methods" (CCSSI 2010, p. 5), and, as the introduction to CCSSM goes on to say, the order of the Standards neither implies a teaching sequence nor sets out the connections among ideas in different topics.

Therefore, what we make of CCSSM will matter—how we keep, or fail to keep, the deep learning of every student at the center of our work. Implementing a set of standards—that, like any set of standards, is necessarily flawed—will require skepticism and courage to support all our students to be mathematics learners and thinkers in ways that will serve them well beyond K–grade 12.

I wrote to a dozen or so of the most experienced classroom teachers I know, inquiring about what issues CCSSM raises for them. All of them have a particular focus on the teaching

and learning of mathematics in their practice and, while still in the classroom, are also active in writing about teaching and in leading professional development. Their responses touched on many issues—from over-testing students, to Standards that they think have been pushed too far down to the lower grades, to dwindling support for sustained, long-term professional development. Two responses stood out:

1. Will all students be engaged in significant mathematics? In particular, will students be given time to develop foundational understandings, or will they be pushed to memorize procedures and definitions so that they can pass a test? One teacher expressed her concern that students, especially students who have some difficulty in mathematics, will be "hurried along a learning trajectory, 'being remediated' without thought given to developing number sense and a strong foundation in the mathematics."

2. Will the Standards for Mathematical Practice be taken seriously as Standards? Will they be integrated into instruction with intention and focus? As one teacher said, "I'm concerned that the mathematical practices will get pushed aside, . . . that the assessments that are developed won't assess those practices."

These two concerns are linked. The Standards for Mathematical Practice focus on what it means to do mathematics. They portray aspects of mathematics that invite students into a living, creative, engaging discipline. Integrating the Practice Standards (and notice that these *are* standards) with core content is one way to mine CCSSM for ways in which its standards can serve all students.

Constellations of content and practice

If the Standards for Mathematical Practice are taken seriously, we must focus on them in the same way we focus on any other standards—with targeted, intentional, planned instruction. Already the place of the Practice Standards in the classroom is being undermined by superficial approaches that boil down to "we are doing all the Practices all the time." Is it possible to be engaged in all the Practices in any one activity? Perhaps. But if students are to learn about *how* to engage in these Practices, actual instruction must be devoted to them. We would not be satisfied to say that we are "doing place value" or "doing measurement" all the time and therefore do not need to devote particular class sessions to these topics. If these Practices are happening "all the time," the result will be that none of them are happening with any attention or depth. If they are only listed on the wall, they will soon be treated like wallpaper and ignored.

Yet we do not want them relegated to special sessions apart from core mathematical content; they are necessarily embedded in content. If students at a particular grade level are learning to use precise language (Practice 6), they are developing that language *about* mathematical ideas they are trying to articulate. If they are learning to make mathematical arguments (Practice 3), they are making these arguments in the context of particular mathematical content that is both accessible to them and significant for their learning. Therefore, my recommendation is this: At each grade level, we must identify content in the curriculum where a teaching-learning emphasis on each Practice can most productively occur—let's call these *Content-Practice nodes*. Work on a particular Practice is not confined to these nodes—once introduced, the Standards of Practice will certainly continue to come up, just as ideas about measurement, once worked on, may come up in other contexts—but these Content-Practice nodes are the places where we can build instructional focus on specific *constellations* of Content Standards and Practice Standards.

During the past few years, I and my colleagues Deborah Schifter and Virginia Bastable have been working on content that connects arithmetic and algebra, which has suggested to us possibilities for constellations of Content and Practice Standards (Russell, Schifter, and Bastable 2011a; Russell, Schifter, and Bastable 2011b; Schifter, Russell, and Bastable 2009). In particular, CCSSM emphasizes a focus on understanding "properties of the operations" and "relationships between operations" in the elementary grades. This emphasis on understanding and using the way each operation behaves as part of the underpinnings of

computation opens up important territory for an instructional focus on three of the Standards of Practice. I name them in this order to reflect how they come up in the next section:

1. Practice 8: "Look for and express regularity in repeated reasoning"
2. Practice 6: "Attend to precision"
3. Practice 3: "Construct viable arguments and critique the reasoning of others"

In the following example from the classroom of a third-grade teacher involved in this work, the practices above are an explicit focus of instruction in the context of students' work on the properties and behaviors of the operations.

Expressing regularity, attending to precision, and making mathematical arguments

In Olivia Miller's third-grade classroom, students are working on equivalent addition expressions. They start by noticing and describing the regularity in pairs of expressions, such as the following:

29 + 37	52 + 49
30 + 36	51 + 50

After a few sessions, students articulate some of their ideas about the relationship between the expressions in each pair. Using the students' own language, the teacher records what they have to say:

- "We can change the numbers but still have the same answer."
- "The numbers can go up and down."
- "We change the numbers by making one less and the other one bigger."
- "We can take away one and then add one."

Individual students discuss what they mean by each of their assertions, and the teacher uses this discussion as an opportunity to remind students about mathematics vocabulary that they know, to ask them to clarify which operations they have in mind, and to challenge them to express their emerging ideas with greater precision. Now the students' collective work reads as follows:

> When we have an addition expression, we can change the numbers but still have the same answer (sum). The numbers can go up and down. We change the numbers by making one less and the other one bigger. We can take away one from one of the addends and then add one to the other addend.

As the class's ideas progress, students bring up the idea that amounts other than one can be added and subtracted from the two addends without changing the sum. Carl states, "We could switch a three like in 30 + 53 and 33 + 50."

Miller replies, "So, I'm wondering, can you 'switch' *any* amount around, or does it just have to be a one?"

Student responses vary:

- "One."
- "Maybe other numbers."

Understanding and applying the practice standards

Some Common Core State Standards of Practice for Mathematics (CCSSI 2010, pp. 6–8) are more general than others. It is possible to argue, for example, that Practice 1—"Make sense of problems and persevere in solving them" (p. 6)—should be happening *all the time*. However, carefully analyzing this Standard uncovers components that require explicit attention; for example, "try special cases and simpler forms of the original problem."

As we work out nodes of instruction for the Standards of Practice (below), some may require more extensive and explicit attention than others.

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

- “I think a two will work.”
- “I don’t think it has to be just a one, but I’m not sure why.”

The class returns to this idea periodically. Miller supports her students in looking for regularities in further examples; working toward describing their conjectures clearly; and finding ways to prove their ideas by using models, diagrams, or story contexts. A few sessions later, some students are ready to present arguments that if you add an amount to one addend and subtract that amount from another addend, the sum remains the same. Anthony holds up a stick of forty-nine connecting cubes and another stick of twenty-three cubes.

Anthony: My smaller stick is twenty-three, so I take one cube, and I make it twenty-four and then subtract another one and give it to the twenty-four, and now I have twenty-five. At first I started with twenty-three, and I’m making my other piece get bigger. So, if you have a small amount, I can move cubes around and make the other stick bigger. [*See* **fig. 1**.]
Miller: Can someone explain what Anthony did?
Eduard: He is taking away from the bigger number and adding it to the smaller number. If you take away something, one of the numbers is getting bigger.
Jasmine: It’s like we are moving the one around from one number to the other, and it doesn’t really change anything. The numbers change, but I don’t think the answer does.

Anthony begins his argument using specific numbers for one of the addends, but the conversation moves quickly into more general terms: “If you take away *something*, one of the *numbers* is getting *bigger*.” Miller builds on Anthony’s and Eduard’s moves toward thinking of the sticks of cubes as representing any amount by explicitly asking her students about this:

Miller: I don’t know how many cubes Anthony has because it is difficult for me to count them. One stick is really long. Do we need to know the problem he is working with?
Liana: We know it’s forty-nine and twenty-three because he told us, but we don’t need to really know. He isn’t adding anything new; he is just getting it from the big stick.
Karim: He is taking away from the big stick and giving it to the little stick.
Serina: We are changing the numbers by some number, sometimes one or two or maybe even ten.
Darin: We could even take away the whole big stick and give it to the little stick and have no second stick. [*See* **fig. 2**.]

Throughout these lessons, students are learning what it means to investigate a regularity (Practice 8), articulate ideas and conjectures about the regularity (Practice 6), and develop arguments to support their conjectures (Practice 3). The class is gradually moving toward developing a complete mathematical argument for a well-articulated rule.

Miller deliberately structures the sessions around regularities that are based on the properties of operations. One way to describe what the students are working on is as an application

FIGURE 1

A *representation-based proof* is what the authors call an argument that young students develop based on a drawing, model, or story context. For the definition, criteria, and examples of representation-based proof, see Russell, Schifter, and Bastable (2011a).

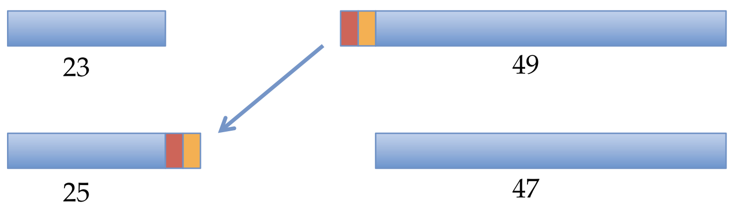
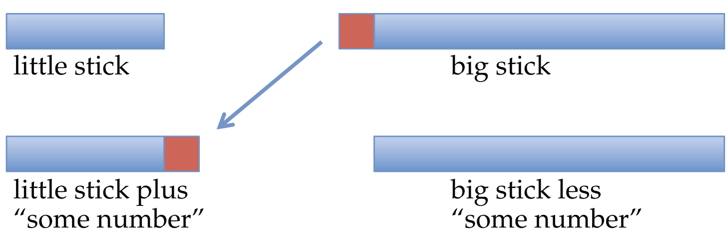


FIGURE 2

In his argument, Anthony used sticks to demonstrate that when adding an amount to one addend and subtracting that amount from another addend, the sum remains the same.



of the associative property of addition. That is, to see why 29 + 37 = 30 + 36, we can apply the associative property:

$$29 + 37 = 29 + (1 + 36) = (29 + 1) + 36 = 30 + 36$$

—or more generally, $a + (b + c) = (a + b) + c$. Because considering the behavior of an operation *in general* and *across multiple problems* is new to Miller's students, she begins this work in the familiar territory of addition and uses numbers with which they can easily compute. For these discussions, she does not want students to focus on difficult computations but on the behavior of the operation. In grade 3, one of the CCSSM Content Standards (3.NBT.2) reads:

> Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. (p. 24)

Once students have worked with this generalization in the context of smaller numbers, they will apply it to problems with greater numbers. Soon after the sessions recounted here, a student remarked, "And you can do it with hundreds, too."

As the year goes on, Miller will also build these investigations of the properties of the operations into the class's work on multiplication and division to connect with, for example, 3.OA.5: "Apply properties of operations as strategies to multiply and divide."

Working in classrooms around the country, my colleagues and I have seen consistently that through such investigations, students become accustomed to noticing regularities and articulating and justifying general claims about them. Once a generalization has been established and justified, students often raise questions about whether their rule applies to other operations. By comparing operations, they establish how each operation has its own set of behaviors.

Skepticism and courage

What are our responsibility and response in the CCSSM era? The worst possible response is fear. That response, dominating some of the discussions I am hearing in schools, can lead only to implementations and assessments of CCSSM that treat these Standards as a list to be learned, ignore the need to weave a coherent course of instruction onto its framework, and put in place strategies designed only to get students through the next test rather than to build reliable concepts and skills.

Miller's class is in a high-poverty school that has performed poorly on state tests, has failed to show adequate yearly progress in some years, and has recently mandated that some instructional time in mathematics be devoted to a computerized instruction-assessment system. Miller confesses,

> I do take more time than I am "allowed" on many lessons when I feel the students will benefit from this. But then I always worry that I will be questioned because I am not on pace. I try to make the best decisions for my students, though.

What is in our future as CCSSM is implemented? Will all students be supported to do the kind of significant mathematics learning that we glimpsed in Miller's class? Will rich, focused instruction build on students' intelligence and enhance student engagement by putting the opportunity to think at the center of math lessons? Will Miller, and all teachers, be supported to develop foundational understandings and practices in their classrooms? Or will she and her students find themselves in a system geared to "covering" the Standards and passing the next test?

CCSSM does not tell us how to teach. It offers a framework that must be interpreted and implemented using all the knowledge about children's varying learning needs and strengths that we, as educators, bring to our work. One way to do this is to take seriously the backbone of CCSSM—the Standards for Mathematical Practice—and to weave them into the content at each grade level, with deliberation and focus, by developing constellations of Content and Practice Standards. This approach has the potential to put mathematical thinking at the heart of the math lesson and to build, for all students, a solid, lasting mathematical foundation. Teaching in the era of the Common Core State Standards for Mathematics requires skepticism and courage—skepticism that any set of standards provides an adequate description of instruction or "solves" the complex issues of mathematics teaching and learning, and courage to take the time to adequately develop foundational ideas of both mathematical content and practices to support entrance into mathematics as a discipline for all students.

This article is based on work funded in part by the National Science Foundation (NSF), grant nos. ESI-0550176 and DRL-1019482. Any opinions, findings, conclusions, or recommendations expressed here are those of the authors and do not necessarily reflect the views of the NSF. Teacher and student names in this article are pseudonyms.

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